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A data driven approach for coarse-graining Stokes-Darcy systems

Ramansh Sharma

T1 - Physics and Chemistry of Materials

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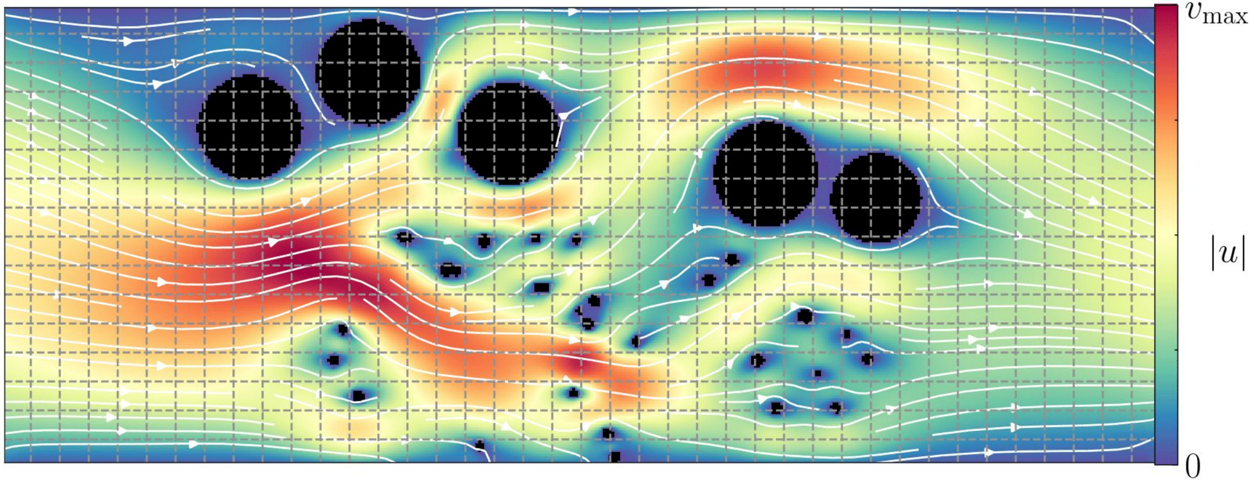
Introduction



Study of flow through porous media is of great interest with applications in

- Petroleum engineering
- Groundwater hydrology
- Environmental science

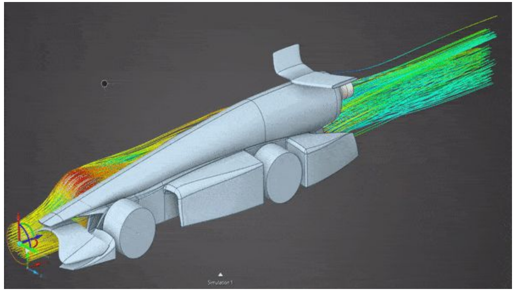
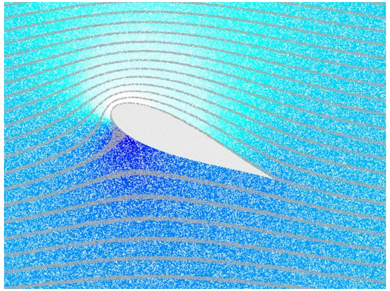
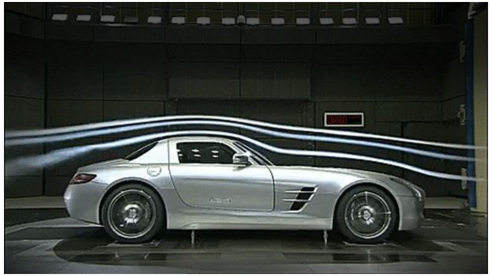
Incompressible Navier-Stokes equations

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = -\nabla p + \mu\nabla^2\mathbf{u} + \mathbf{f}$$

$$\nabla\cdot\mathbf{u} = 0$$

Reynolds number

$$\mathrm{Re} = \frac{\rho u L}{\mu}$$

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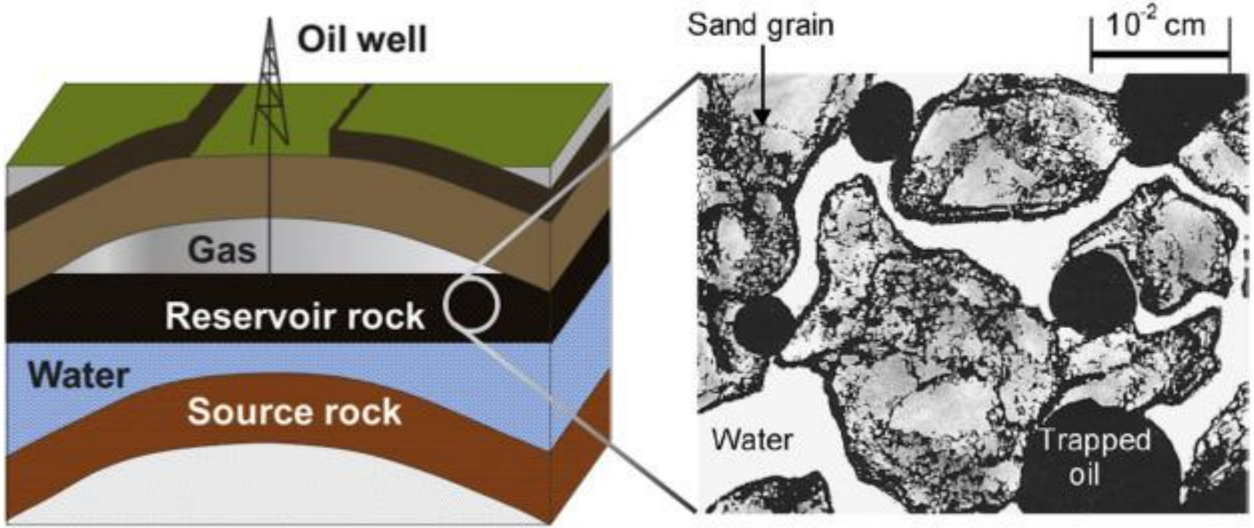
Application of porous media flow



We are interested in modeling flow at very low Reynolds numbers (Re << 1), aka creeping flow.

Image reference: Perazzo, Antonio, et al. "Emulsions in porous media: From single droplet behavior to applications for oil recovery." *Advances in colloid and interface science* 256 (2018): 305-325.

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Stokes flow

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = -\nabla p + \mu\nabla^2\mathbf{u} + \mathbf{f}$$

$$\nabla\cdot\mathbf{u} = 0$$

Expensive to solve on a fully resolved geometry!

Solution: coarse graining.

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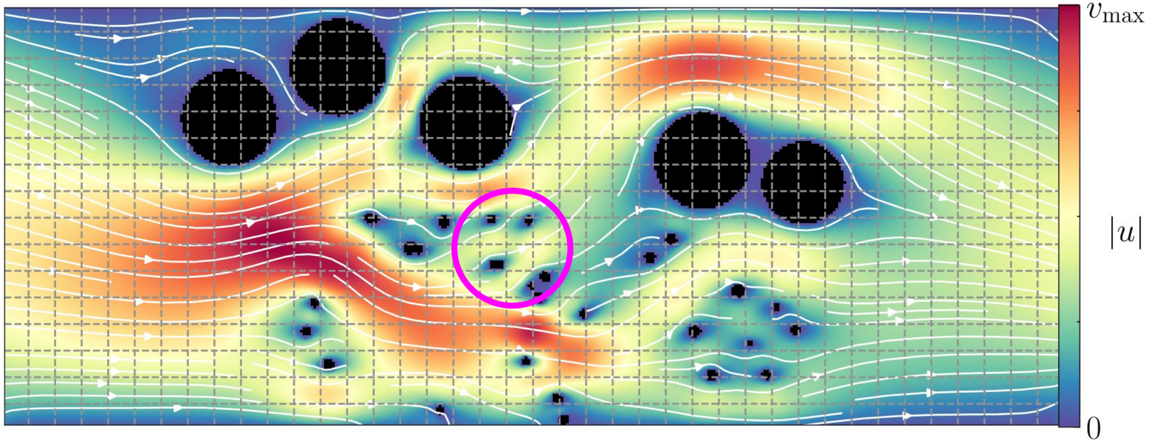
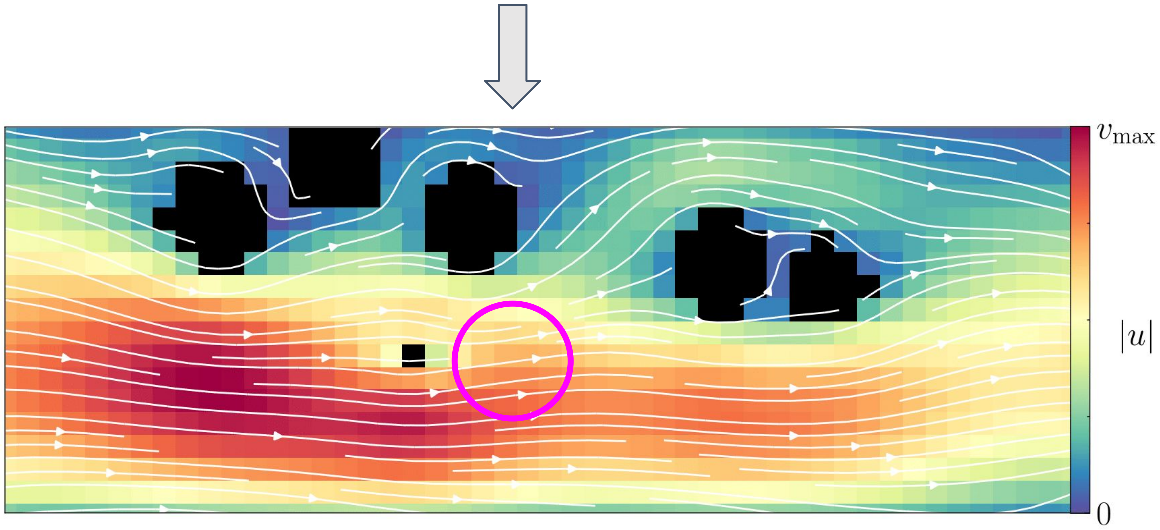
Problem

Pores become unresolved at the coarse scale.



One **cannot** naively solve Stokes flow at coarse scale.



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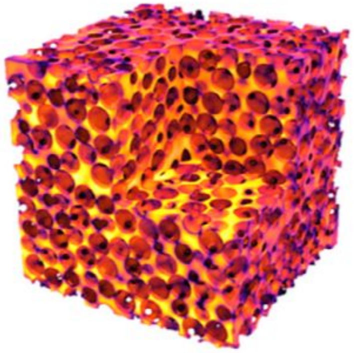
Past work

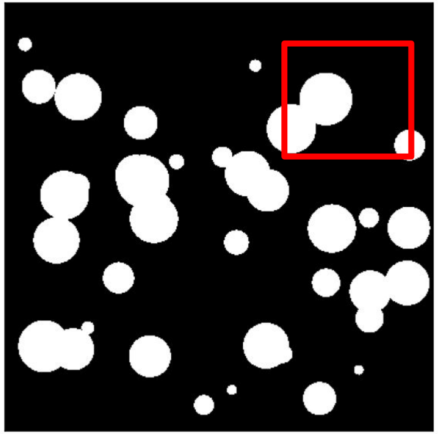
Related LANL work

Santos, Javier E., et al. "PoreFlow-Net: A 3D convolutional neural network to predict fluid flow through porous media." 2020.

Santos, Javier E., et al. "Computationally efficient multiscale neural networks applied to fluid flow in complex 3D porous media." 2021.



Velocity Field



The phenomenological effect of the porous medium is mostly localized!

Brinkman equations

Bridges Darcy's law (**coarse scale**) and Stokes flow (**fine scale**).

$$\mu \Delta \mathbf{u} - \nabla p - \alpha(\mathbf{x})\mathbf{u} = \mathbf{0},$$
$$\nabla \cdot \mathbf{u} = 0,$$

Stokes: $\mu \Delta \mathbf{u} - \nabla p$

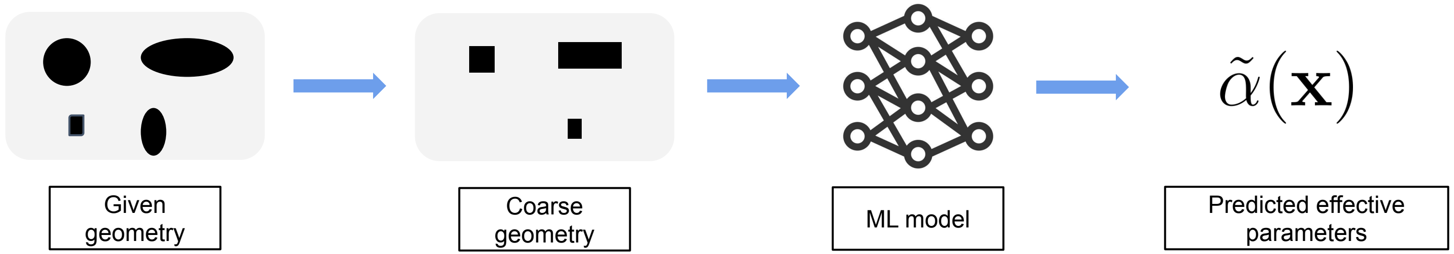
Darcy: $- \alpha(\mathbf{x})\mathbf{u}$

Effective parameters: $\alpha(\mathbf{x})$ - Empirical resistivity field (**always positive**)

Key insight: At the coarse scale where geometry is unresolved, we model the medium using **effective parameters** that approximate the impact of the porous geometry.

Proposed data-driven workflow

The goal is to use machine learning to infer the spatially varying resistivity field from a given geometry.



To achieve this we need to form a dataset of a large number of porous geometries and the corresponding restivitity fields.

Direct numerical solver

The numerical solver uses a staggered grid and central finite-differences with a second order accurate scheme for the velocity and first order accurate scheme for the pressure. The component wise Brinkman equations are,

$$
\begin{aligned}
\mu \Delta u - \alpha_u(x,y)\, u - \frac{\partial p}{\partial x} &= 0, \\
\mu \Delta v - \alpha_v(x,y)\, v - \frac{\partial p}{\partial v} &= 0, \\
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0,
\end{aligned}
$$

where **u** = (u, v) is the velocity, α_u and α_v are the resistivity parameters at the MAC grid cell faces. The *sparse* block linear system that emerges is,

$$
\underbrace{\begin{bmatrix} L_u & \mathbf{0} & G_x \\ \mathbf{0} & L_v & G_y \\ D_x & D_y & \mathbf{0} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} u \\ v \\ p \end{bmatrix}}_{\mathbf{y}} = \mathbf{0},
$$

where

$$
\begin{aligned}
L_u &= (\Delta - \alpha_u)\,\cdot \\
L_v &= (\Delta - \alpha_v)\,\cdot
\end{aligned}
$$

Dataset generation

current stage of project

The **differentiable solver** algorithm to optimize for the coarse scale effective parameters.

Initialize $\theta = [\alpha(\mathbf{x})]$

Until convergence:

Form $\underbrace{\begin{bmatrix} L_u & \mathbf{0} & G_x \\ \mathbf{0} & L_v & G_y \\ D_x & D_y & \mathbf{0} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} u \\ v \\ p \end{bmatrix}}_{\mathbf{x}} = \mathbf{0}$

Solve $A_\theta \mathbf{y} = \mathbf{0}$

Compute $L = \|y - y_{\text{gt}}\|_2$

Update θ

Coarsened true solution

We run the solver for a large number of porous media geometries.

We manually provide the jacobian $\frac{\partial L}{\partial \theta} = -\left(\frac{\partial L}{\partial y}\right)^T A^{-1} \frac{\partial A}{\partial \theta} y.$

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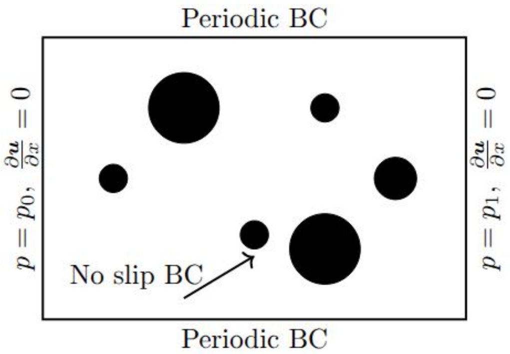
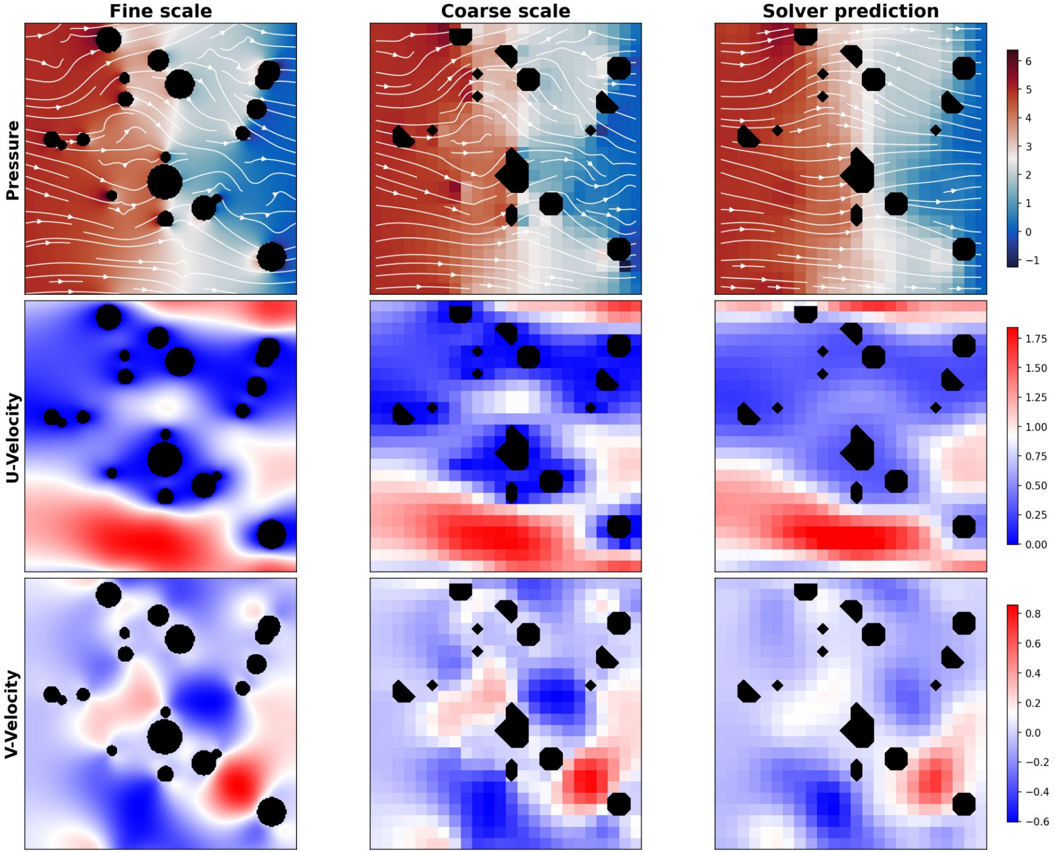
Differentiable solver example

Solver details



Relative ℓ_2 errors

pressure : 0.077
u-vel : 0.301
v-vel : 0.421



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Conclusion

Summary:

- Modeling fluid flow through porous media efficiently and accurately is relevant for a wide range of applications, but doing so numerically is challenging.
- We take first steps towards developing a method to solving the inverse problem of predicting the phenomenological parameters of the Stokes-Darcy equations.

Next steps:

- Train different ML techniques to learn a map from geometry to the effective parameters.

Thank you! Questions?

Big thank you to mentors: Alessandro Gabbana, Kipton Barros, Agnese Marcato, Javier Santos, Daniel Livescu, Varun Shankar.