Fluids You Can Trust: Property-Preserving Operator Learning for Incompressible Flows

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Research Problem

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = \nu \nabla^2 \mathbf{u} - \frac{1}{\rho}\nabla p + \frac{1}{\rho}\mathbf{f},$$
$$\nabla \cdot \mathbf{u} = 0.$$

Can operator learning for incompressible flows be made both accurate and physically trustworthy?

Why this matters

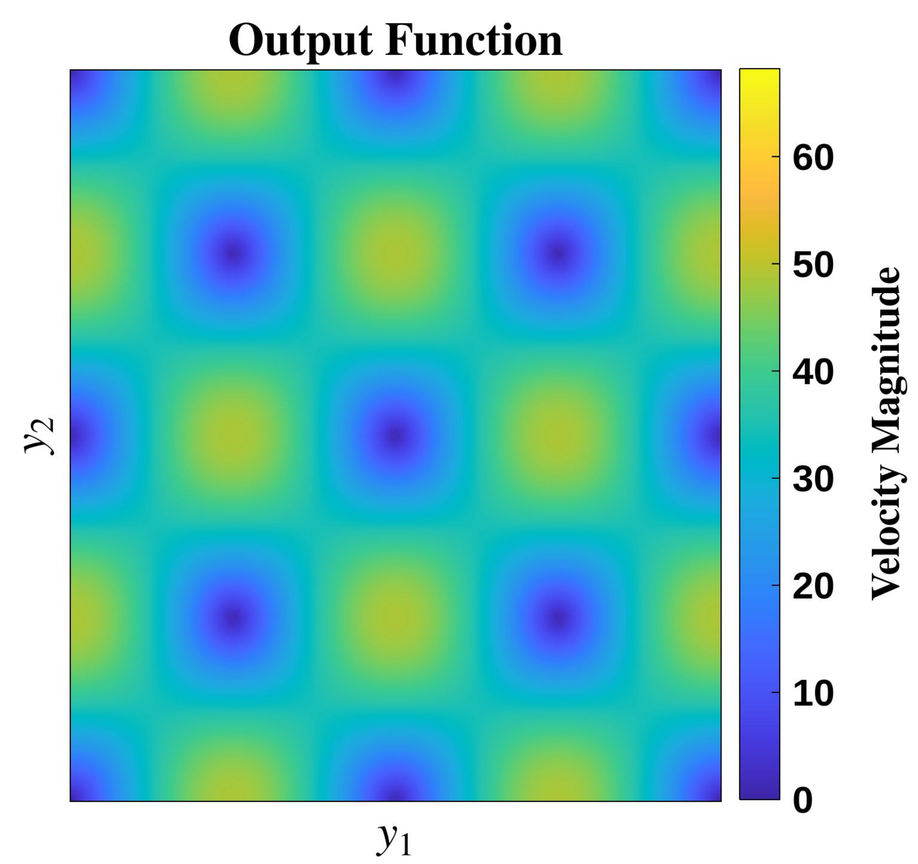
- Traditional incompressible numerical solvers are computationally expensive.
- Modern neural operators are scalable and accurate, but typically cannot exactly preserve physical properties.
- For incompressible flows, this produces predictions that are physically inconsistent.

Method

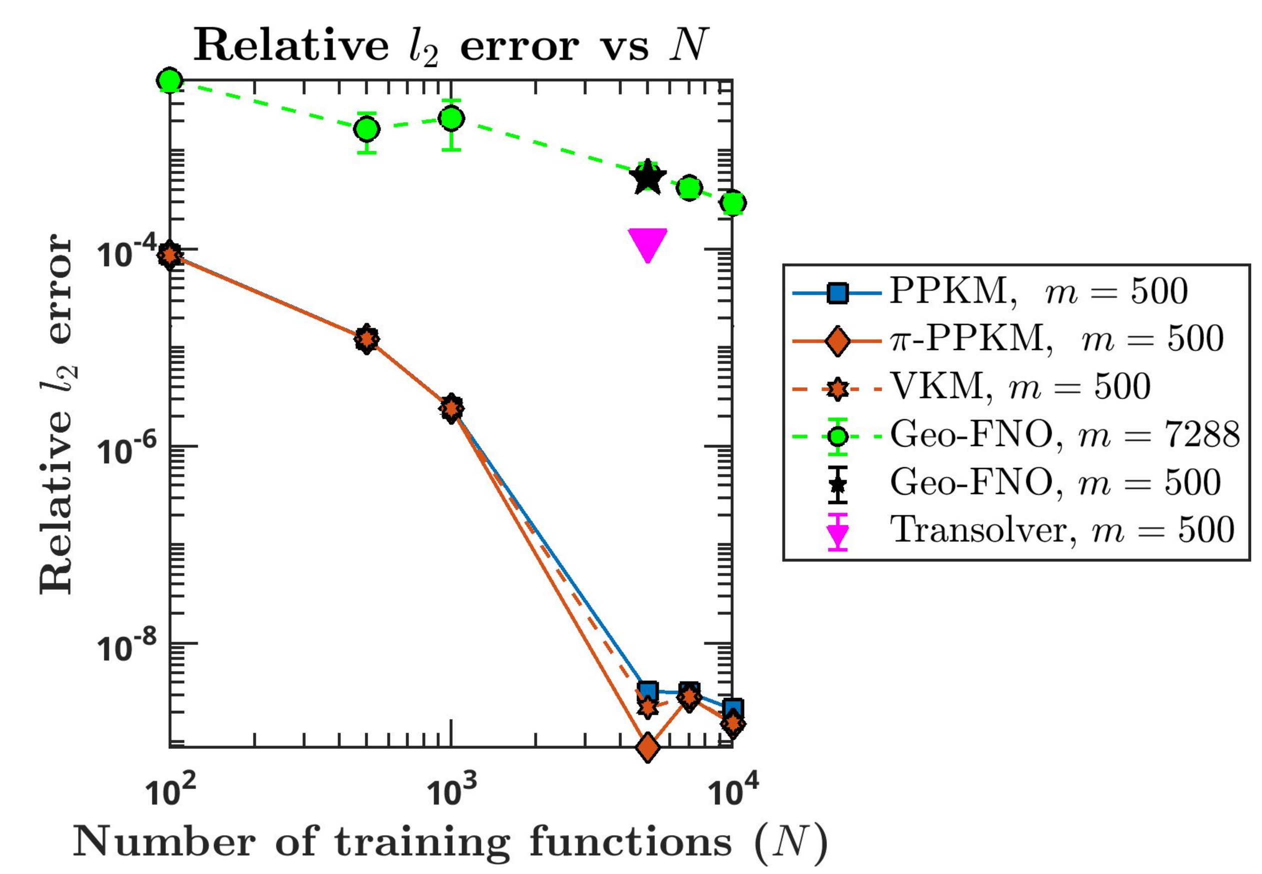
- We learn the operator $\mathcal{G} : \mathcal{A} \to \mathcal{V}$, where the output functions are incompressible velocity fields (with potentially additional properties).
- Our property-preserving kernel method (**PPKM**) uses a matrix-valued kernel Φ to interpolate the output functions.
- The operator kernel Λ is used to map the input functions to the spatial interpolation coefficients.
- **Key idea**: By mapping the input functions to the output interpolation coefficients, the final prediction analytically preserves the properties of Φ: incompressibility, periodicity, turbulence power laws.

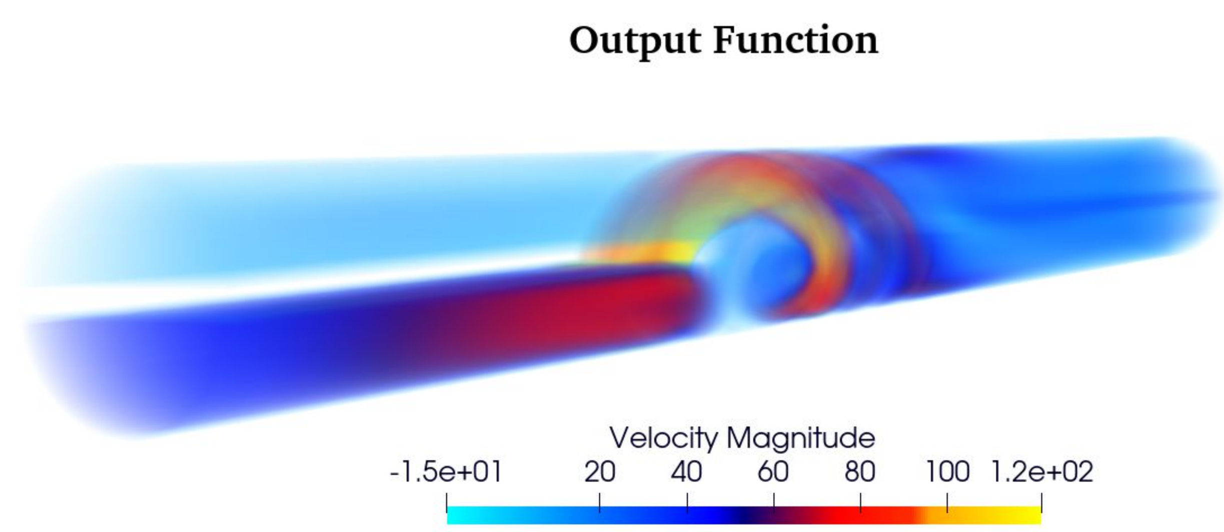
Results

We compared **PPKM** against the vanilla kernel method (**VKM**), Geo-FNO, and Transolver on a suite of 2D and 3D incompressible flow problems.

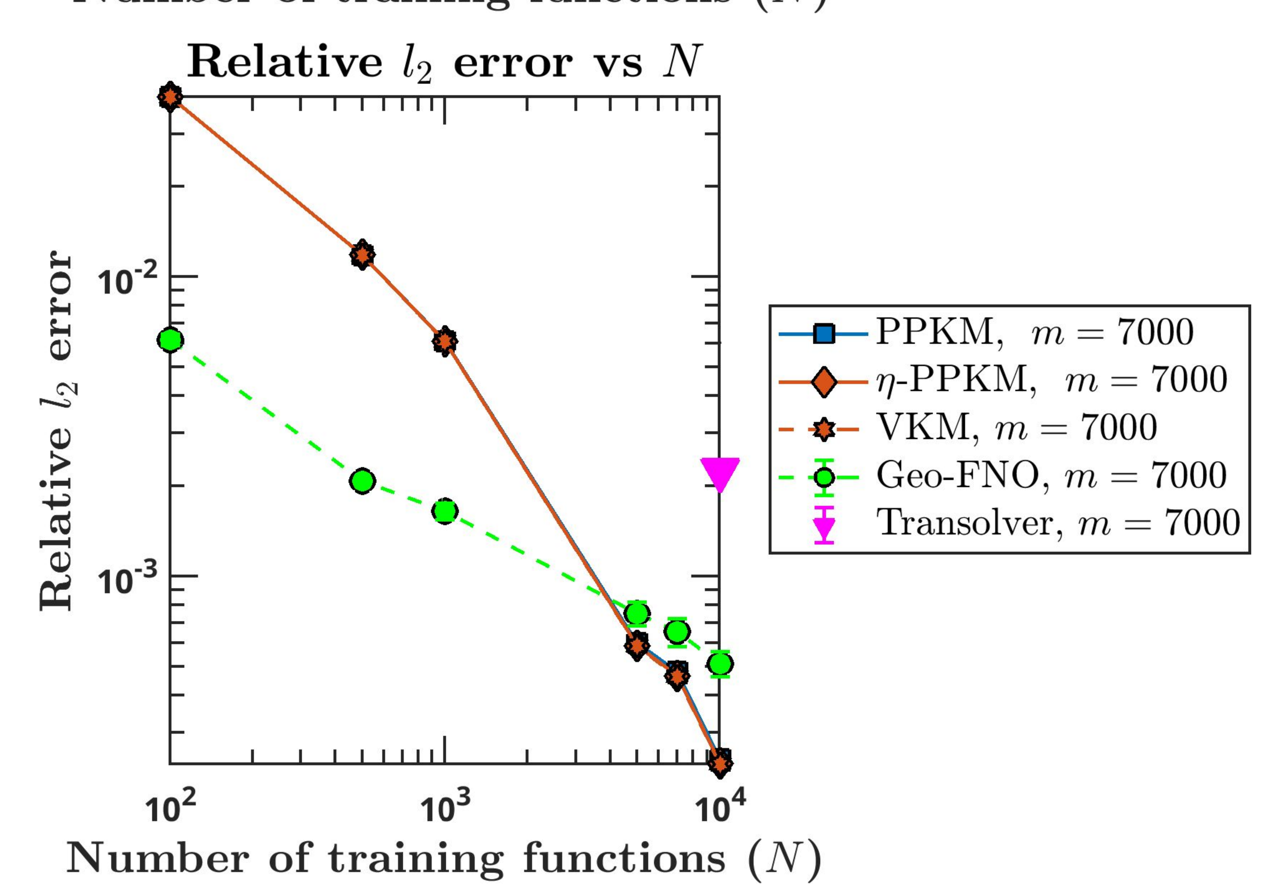


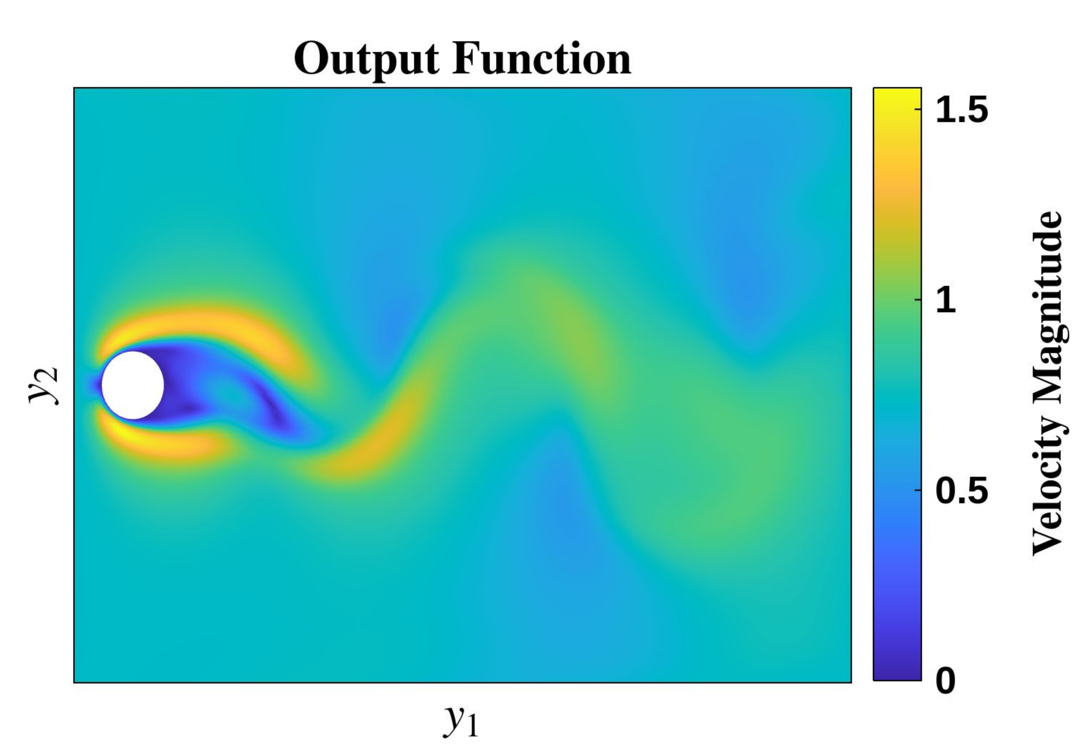
2D Taylor-Green Vortices



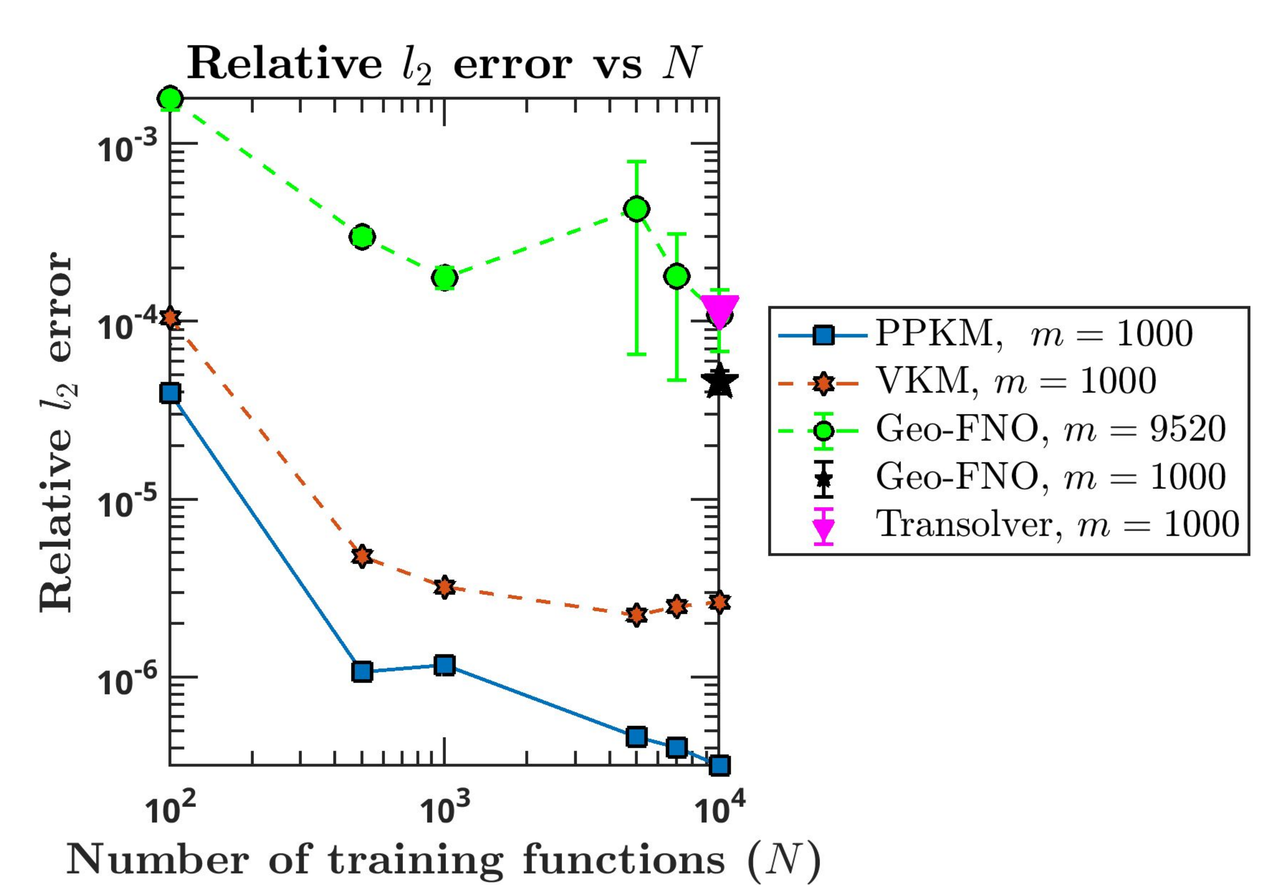


3D Species Transport





2D Flow Past a Cylinder



Conclusion

We introduce the **PPKM** which can analytically enforce incompressibility, periodicity, and turbulence power laws. Some of its advantages over neural operators are:

- **1-6** orders of magnitudes lower relative errors.
- **5** orders of magnitudes faster training.
- Fewer training parameters (only **2** in **PPKM**).
- Natural extension for uncertainty quantification.

Future work

- Sparse kernel matrices to reduce computational cost.
- Explore localized kernel methods over the current global approach.
- Extend to electromagnetism and magnetohydrodynamics.

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